

A LOT OF FLAVOR PHYSICS FROM A LITTLE SYMMETRY*

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Recent neutrino parameter measurements place increasingly stringent constraints on acceptable supersymmetric theories of flavor. A very fruitful approach is the application of non-Abelian discrete gauged flavor symmetries G_f . We discuss a highly successful model using $G_f = T' \times Z_3$, where T' is a non-Abelian group based on the symmetries of a tetrahedron. This model reproduces the basic features of the $U(2)$ model, and also predicts neutrino masses and mixing angles consistent with either the SMA or LMA scenarios.

The family replication problem and the strongly hierarchical pattern of fermion masses and mixing angles has remained one of the more intractable problems in particle physics. Any acceptable flavor theory must explain why, for example, $m_t/m_e \approx 350,000$ while $|V_{ub}| \approx 3 \times 10^{-3}$. The situation has become even more interesting with the observation of nonzero neutrino masses and mixings¹ that do not appear to follow the same hierarchies as in the charged fermion sector. Indeed, the combined data of existing experiments (omitting LSND²) suggests a large mixing angle due to the atmospheric ν deficit,

$$\sin^2 2\theta_{23} > 0.8, \quad 10^{-3} \text{eV}^2 \leq \Delta m_{23}^2 \leq 10^{-2} \text{eV}^2, \quad (1)$$

and a solar ν deficit explained by either the small mixing angle (SMA) MSW solution

$$2 \times 10^{-3} \leq \sin^2 2\theta_{12} \leq 10^{-2}, \quad 4 \times 10^{-6} \text{eV}^2 \leq \Delta m_{12}^2 \leq 10^{-5} \text{eV}^2, \quad (2)$$

or the (now somewhat more favored) large mixing angle (LMA) MSW solution

$$\sin^2 2\theta_{12} > 0.7, \quad 6 \times 10^{-6} \text{eV}^2 \leq \Delta m_{12}^2 \leq 5 \times 10^{-5} \text{eV}^2. \quad (3)$$

with three active neutrino species.

We consider the possibility that a spontaneously-broken horizontal symmetry G_f may explain the totality of fermion mass and mixing angle data. We work in the context of supersymmetry (SUSY), which renders the hierarchy between the scale of flavor physics and the weak scale stable against radiative corrections.

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Flavor-changing neutral current (FCNC) effects, which are present in generic SUSY models, can be brought under control by the same symmetry G_f if it leads to a near mass degeneracy between the first two generations of squarks or sleptons in the low-energy effective theory. We require this to be the case.

We advocate the study of discrete gauge symmetries for G_f . It has long been argued³ that global symmetries are violated by quantum gravity effects and may therefore be inconsistent as fundamental symmetries of nature. Alternately, continuous gauge symmetries in the context of SUSY tend to give rise to excessive FCNC's through D-term interactions.⁴ Discrete gauge symmetries avoid both of these pitfalls. Theories with global and gauged discrete symmetries differ in that there is information contained in topological defects found only in the latter.^{5,6} For model-building purposes, the relevant low-energy constraint for discrete gauge symmetries is an anomaly cancellation condition linear in G_f and quadratic in non-Abelian Standard Model gauge groups.⁷ Our model satisfies this condition.

The central feature of discrete groups is a finite number of inequivalent irreducible matrix representations (reps). Discrete groups tend to have a rich representation structure, which opens many possibilities for assigning quantum numbers in model building. Reps of dimension >1 appear only in non-Abelian groups. In particular, doublets are used to impose the near-degeneracy of sparticle masses in the first two generations, while the large value of m_t suggests that the third generation is distinguished: a $\mathbf{2} \oplus \mathbf{1}$ structure.

A model⁸ based on the continuous group $G_f = U(2)$ is particularly successful in reproducing all observed charged fermion masses and mixings, especially when combined with an $SU(5)$ GUT. The Yukawa matrix textures are given by

$$Y_D \sim Y_L \sim \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & \epsilon & \epsilon \\ 0 & \epsilon & 1 \end{pmatrix}, \quad Y_U \sim \begin{pmatrix} 0 & \epsilon'\epsilon & 0 \\ -\epsilon'\epsilon & \epsilon^2 & \epsilon \\ 0 & \epsilon & 1 \end{pmatrix}, \quad (4)$$

where the symmetry is broken at two scales, $U(2) \xrightarrow{\epsilon} U(1) \xrightarrow{\epsilon'} \text{nothing}$, with $\epsilon \approx 0.02$ and $\epsilon' \approx 0.004$. In order to produce these textures, the $U(2)$ model requires $\mathbf{1}$, $\mathbf{2}$, and $\mathbf{3}$ reps and the multiplication rule $\mathbf{2} \otimes \mathbf{2} = \mathbf{3} \oplus \mathbf{1}$.

One finds⁹ that the smallest group satisfying the same conditions is T' , the group of proper rotations leaving a regular tetrahedron invariant in the $SU(2)$ double covering of $SO(3)$. Moreover, including an extra Z_3 factor, so that $G_f = T' \times Z_3$, is the minimal extension needed to reproduce $U(2)$ model textures and satisfy discrete anomaly cancellation conditions. Additionally, the rich representation structure of T' allows for neutrinos to be placed in different reps than the charged fermions, which is the origin of different hierarchies in the two sectors. The symmetry breaking is $T' \times Z_3 \xrightarrow{\epsilon} Z_3^{\text{diag}} \xrightarrow{\epsilon'} \text{nothing}$, and with the charge assignments presented in⁹, the light neutrino mass matrix (via the seesaw mechanism) is

$$M_{LL} \sim \begin{pmatrix} (\epsilon'/\epsilon)^2 & \epsilon'/\epsilon & \epsilon'/\epsilon \\ \epsilon'/\epsilon & 1 & 1 \\ \epsilon'/\epsilon & 1 & 1 \end{pmatrix}. \quad (5)$$

Table 1. Data and fit values for the LMA scenario in the $T' \times Z_3$ model (ν sector).

| Observable | Expt. value | Fit value |
|-------------------------------------|-------------|-----------|
| $\Delta m_{23}^2 / \Delta m_{12}^2$ | 20 – 1670 | 376 |
| $\sin^2 2\theta_{12}$ | > 0.8 | 0.88 |
| $\sin^2 2\theta_{23}$ | > 0.8 | 0.83 |
| $\sin^2 2\theta_{13}$ | < 0.18 | 0.10 |

The 23 mixing angle is clearly $O(1)$; since $\epsilon'/\epsilon = O(1/5)$ is neither especially large nor small, adjustment of the implicit $O(1)$ coefficients characteristic of an effective field theory allows for either LMA or SMA solutions for θ_{12} .

We present in Table 1 a fit to the observable neutrino mass and mixing parameters for LMA; the full fit also includes the charged fermion parameters, but is excellent and hence omitted for brevity. A similarly excellent SMA fit appears in ¹⁰. The unknown $O(1)$ coefficients c_i are constrained to lie approximately in the range $1/r \leq |c_i| \leq r$ by adding terms to the χ^2 function of form $\Delta\chi_i^2 = (\ln |c_i| / \ln r)^2$; for this fit, $r = 3$. One finds $\chi_{\min}^2 = 11.8$, smaller than the total number of fermion observables (16), which we argue¹⁰ is the figure of merit.

The explosion of neutrino data and better bounds on rare FCNC processes will provide ever more precise clues as to the nature of flavor physics. We have seen how well a rather small discrete gauge group can accommodate all of the currently measured observables and anticipate the refinement of such models in the future.

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